

## OPTIMUM PORT NUMBERING IN THE ELECTROMAGNETIC SIMULATION OF COMPLEX NETWORKS

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### ABSTRACT

A new strategy based on combinatorial optimization of the port numbering in combination with the Mode Matching method has been developed for the fullwave analysis of complex networks. The approach has been implemented on both serial and parallel platforms and is shown to provide excellent efficiency (more than 20 times of speed-up) overcoming the limitations of previous approaches based on graph theory.

### INTRODUCTION

In order to avoid trimming and tuning of the circuit realized The use of electromagnetic (EM) design tools based on rigorous numerical methods is of paramount importance or even indispensable in the design of microwave circuits. When complex circuits have to be designed, the numerical effort involved by EM models, may easily become unaffordable.

Among the possible numerical methods, we consider here the Mode Matching (MM) technique, as one of the best performing approaches for the rigorous modelling of microwave circuits [1-5]. In spite of its efficiency, however, the MM analysis and, particularly, the design of complex networks, needs to be associated with suitable numerical strategies in order to keep the numerical expenditure within affordable limits.

The MM implementation we refer here is based on the Generalized Admittance Matrix (GAM) approach [1-3]. After segmenting the network into

elementary cells, the GAM of each elementary cell or discontinuity is computed, then GAMs of all cells are combined to generate the following global linear system of equations:

$$\begin{bmatrix} \mathbf{I}_e \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{ee} & \mathbf{y}_{ec} \\ \mathbf{y}_{ce} & \mathbf{y}_{cc} \end{bmatrix} \begin{bmatrix} \mathbf{V}_e \\ \mathbf{V}_c \end{bmatrix} \quad (1)$$

where  $\mathbf{I}_e$  and  $\mathbf{V}_e$  are the currents and voltages at the external ports, and  $\mathbf{I}_c$  and  $\mathbf{V}_c$  the currents and voltages at the connected ports. Equating the currents at the connected ports by mean of the equation:  $\Gamma_I \mathbf{I}_c = 0$  we can simplify (1) to obtain:

$$\begin{cases} \Gamma_V \mathbf{V}_c = 0 \\ \mathbf{y}'_{cc} \mathbf{V}_c = -\mathbf{y}'_{ce} \mathbf{V}_e \end{cases} \quad (2)$$

where:  $\mathbf{y}'_{ce} = \Gamma_I \mathbf{y}_{ce}$  and  $\mathbf{y}'_{cc} = \Gamma_I \mathbf{y}_{cc}$ . and:

$$\mathbf{I}_e = \mathbf{y}_{ee} \mathbf{V}_e + \mathbf{y}_{ec} \mathbf{V}_c \quad (3)$$

The matrices  $\Gamma_I$  and  $\Gamma_V$  represent the constraints on voltages and currents imposed by the topology of the circuit.

The solution of the system (1) is obtained in two steps. First we solve the linear system (2) for a given excitation  $\mathbf{V}_e$  at the external ports of the network, then the currents at the external ports are obtained using (3). In the usual applications only the relationship between external variables  $\mathbf{I}_e = \mathbf{Y}_E \mathbf{V}_e$  is required. In a number of cases however also the information on internal voltages (or currents) may be necessary, as for the application of the Adjoint Network Method [3].

In a previous paper [6], it has been shown that the numerical core of the MM simulation is in the solution of the linear system (2). Its performance can be improved substantially by exploiting the characteristic of the system matrix, which is generally sparse, with a zero-percentage normally over 90%. It has also been shown that the  $\mathbf{y}'_{cc}$  matrix pattern depends on the numbering of the physical ports after segmenting the network into

elementary cells. Depending on whether a generally sparse or a banded (skyline) matrix is obtained, very different solution times result. A banded pattern is to be preferred, as banded direct solvers are more efficient than sparse iterative ones. Moreover, since the system matrix is non-definite there is no chance of using direct sparse solvers. The optimum numbering that minimizes the bandwidth of the system matrix, can improve the simulation times of up to 2 orders of magnitude. It should also be reminded that solution times in a banded solver have a quadratic dependence on the matrix bandwidth.

Finding out the optimum port numbering in complex networks is not trivial, as it is an NP-complete problem [7]. In the previous above mentioned paper [6], we have demonstrated that it is equivalent to minimizing the bandwidth of the connection matrix, and we have solved it implementing the modified Reversed Cuthill-McKee (RCM) approach [7]. It is a very efficient graph-theory-based method, we experienced on very large cases such as a 4x4 Butler matrix (Fig. 1). In this paper, limitations and disadvantages of RCM are discussed. An alternative and completely new solution to the problem is then proposed to overcome such limitations. The algorithm has been implemented both on serial and parallel distributed memory RISC platforms.

### OPTIMUM PORT NUMBERING

The Butler matrix represents a typical microwave network used in communication satellite circuits. It has a relatively high degree of complexity, and requires considerable computing resources. It is therefore a good example to use as a test case.

The Butler matrix of Fig. 1 is divided into 2 symmetrical parts. Each part is segmented into 96 elementary cells with a total of 118 ports. An experienced user of the MM simulation code has generally some heuristic rule-of-thumb methods to number the electric ports. In the previous paper [6], we showed results obtained applying RCM to a "heuristically" optimized numbering, demonstrating the substantial efficiency of the method. A further investigation however has identified some important limitations. The first is that the optimized numbering, thus the final bandwidth of the system matrix (1), depend on the starting numbering. The second, and perhaps more substantial limitation is that the RCM has proved

to be ineffective for some initial numberings. In some cases the bandwidth of the final connection matrix is larger than the starting one. An example is shown in Fig. 2. Fig. 2a shows a possible starting matrix corresponding to a chosen initial numbering. Fig. 2b shows the results obtained applying RCM to the matrix of Fig. 2a. It can be observed that the final bandwidth is 30% wider than the starting one. The third and final limitation is that RCM can work only on symmetrix matrices, and this is not appropriate for a future extension of the MM simulation to non-reciprocal devices.

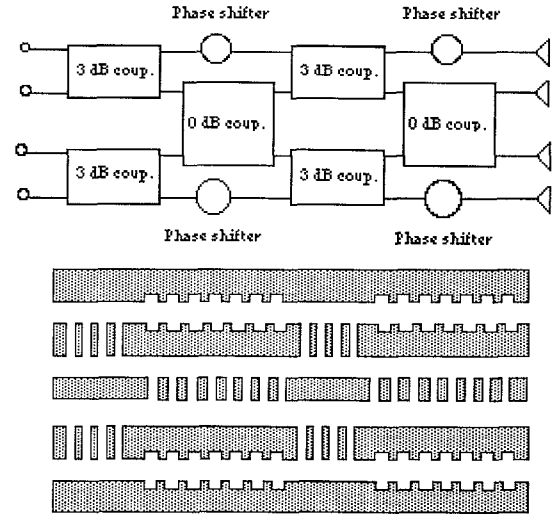


Figure 1: Butler matrix equivalent circuit and tructure

The dependence on the starting numbering is easily explained, since RCM is a heuristic approach, that does not guarantee the global optimization. The RCM failure on some matrices, on the contrary, is a less trivial subject, and studies are running to group all the critical patterns into a certain "class". Anyway, both these limitations prevent the implementation of the approach in an effective CAD MM tool.

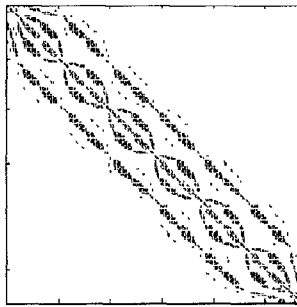
A different approach to the problem is proposed here, based on Tabu Search (TS) [8], a combinatorial optimization method to perform a global search among the possible numberings. This allows one to overcome the sensitivity to starting numberings, and to deal successfully with all the matrix patterns (non-symmetric also).

For the sake of conciseness, the implementation of the method is not described here. The reader is

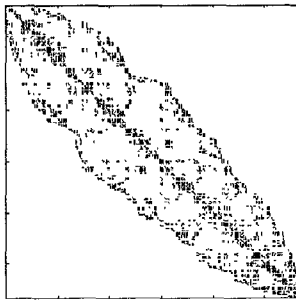
addressed to [8] for a complete and exhaustive description of TS.

## RESULTS

The TS method is costful, from a computational point of view: the optimization of the numbering on the Butler's matrix of Fig. 1 takes, on an IBM 250, about 35 s (average value over a set of 5 cases),



a

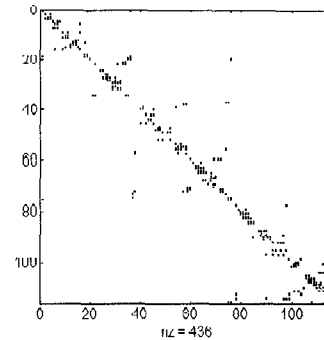


b

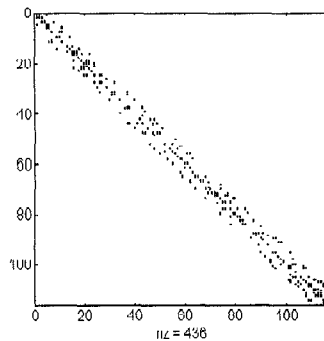
Figure 2: a) Initial connection matrix  
b) RCM optimized connection matrix

instead of the 0.05 s of RCM. This is why a parallel implementation of the TS method becomes useful. TS has been parallelized using the Parallel Virtual Machine (PVM) [9] programming interface, on the IBM SP2 at Perugia University. It is an 8 processor distributed memory platform. A master-slave paradigm has been followed on developing the parallel TS. The computation times for different numbers of processors are shown in Tab. I. Values refer to an average over 5 different starting port numberings. Like in all optimization problems, different levels of optimization can be achieved, depending on the number of steps performed by TS. Times in Tab. I refer to 2000 steps of TS, enough to achieve bandwidths differing less than 5% from

the global optimum. A comparison of data in Tab. I with Ahmdal's law [10] demonstrates the good efficiency and scalability of the implemented algorithm. One of the most interesting results is the following. One of the possible "heuristically optimized" numberings on the Butler's matrix generates the connection matrix of Fig. 3a, with a bandwidth 75. Using the connection matrix of Fig. 3a, the system (1) so generated is solved in 9.5 s on IBM 250 T.



a



b

Figure 2: a) Butler initial connection matrix  
b) Tabu optimized Butler connection matrix

Number of processors	Computing time in sec. (2000 steps)
1	35
2	24.3
4	22.1
6	17.6
7	16.9
8	16.1

Table I

Applying TS to optimize the port numbering, after nearly 40 s (with 8 processors) the global optimum is found, corresponding to the connection matrix in Fig. 3b. Its bandwidth is 6. Generating system (1) with

the so found numbering, the solution time is 0.37 s on IBM 250 T. As the numbering optimization is done once, and does not depend on excitations and frequency, for 100 frequency points the global times are shown in Table II. They refer to three different situations: a "hand-chosen" numbering, a numbering obtained with an intermediate-level optimization, and a fine-level TS optimization. The user can select the desired level by an appropriate choice of the number of TS steps.

Numbering	System Solution Time	Renumbering Time
Hand-tuned	950 s	-
Intermediate TS optimization	48 s	16.1 s
Fine TS Optimization	37 s	40 s

Table II Timing is referred to 100 frequency points and the renumbering is performed on 8 processors

## CONCLUSIONS

The optimum port numbering is important to improve the numerical efficiency of MM analysis. This is of paramount importance when large networks are to be studied. The RCM approach implemented in the past is not appropriate for an effective CAD tool, as it is not always able to optimize the starting numbering. We have demonstrated that a combinatorial approach, based on TS, possibly exploiting a parallel distributed memory architecture (IBM SP2), guarantees an optimization whatever the starting numbering is. Since the numbering is optimized only once, and does not depend on excitations, frequencies, or physical dimensions of the elementary cells, the TS approach to the port renumbering is quite amenable to a cooperative computing environment, where the designer generally uses small platforms (entry level RISC workstations) for the MM analysis, larger parallel platforms being possibly used to find the best numbering. This keeps the computational costs relatively low, as the only numerical core is run on a large platform, where prices for CPU use are higher. In any case, the serial TS optimization times are still good enough for a complete

implementation of the method on a traditional workstation.

Finally, the approach is very general, and can be successfully applied to complex circuits of any kind (not only waveguide circuits) as well as to different simulation techniques in the field of computational electromagnetism.

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